Multiscale Signal Analysis and Modeling

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Editors

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Springer

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ISBN 978-1-4614-4144-1 ISBN 978-1-4614-4145-8 (eBook)
DOI 10.1007/978-1-4614-4145-8
Springer New York Heidelberg Dordrecht London

Library of Congress Control Number: 2012945404

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

In recognition of Dr. Gilbert G. Walter having outlived his prime of 79.

From Xiaoping A. Shen and Ahmed I. Zayed

Preface

This monograph is a collection of chapters authored or coauthored by friends and colleagues of Professor Gilbert Walter in celebration of his 80th birthday. The authors represent a spectrum of disciplines, mathematics, applied mathematics, electrical engineering, and statistics; yet, the monograph has one common theme: multiscale analysis.

Multiscale analysis has recently become a topic of increasing interest because of its important applications, in particular, in analyzing complex systems in which the data behave differently depending upon which scale the data are looked at. The advent of wavelets has given an impetus to multiscale analysis, but other techniques such as sampling and subsampling have been used successfully as a tool in analyzing multiscale signals. For this reason we have decided to include a variety of chapters covering different aspects and applications of multiscale analysis.

The monograph is divided into three main parts: Part I is a collection of chapters on sampling theory while Parts II and III contain chapters on multiscale analysis and statistical analysis, respectively. The level of presentation varies. Few chapters are very specialized, while others are self-contained or of expository nature, but most chapters should be accessible to graduate students in mathematics or electrical engineering.

Part I, which consists of eight chapters, has chapters on sampling, interpolation, and approximation in the space of bandlimited functions, shift-invariant and reproducing-kernel Hilbert spaces, and the Hardy space $H^2(\mathbf{D})$ of analytic functions in the open unit disk. Part II contains four chapters on a unified theory for multiscale analysis, multiscale signal processing, developing algorithms for signal and image classification problems in large data sets, and wavelet analysis of ECG (electrocardiogram) signals. Part III is comprised of three chapters on characterization of continuous probability distributions, Bayesian wavelet shrinkage methods, and multiparameter regularization for the construction of estimators in statistical learning theory.

In Chapter 1, W. Madych revisits the classical cardinal series and considers its symmetric partial sums. He derives necessary and sufficient conditions for its convergence under growth conditions on the coefficients that imply analogous asymptotic behavior of the function represented by the series. Several corollaries, including sampling type theorems, are obtained.

Chapter 2, by F. Stenger, M. Youssef, and J. Niebsch, is also related to the cardinal series and the Sinc function , but in the context of function interpolation and approximation. Function interpolation may be carried out using algebraic polynomials, splines, Fourier polynomials, rational functions, wavelets, or Sinc methods. Function interpolation by one of the above methods frequently has one of the following features: (1) the modulus of the error of approximation near one endpoint of an interval differs considerably from the modulus of the error near the other end-point; (2) the moduli of the errors near the two endpoints of the interval are roughly the same, but differ appreciably from the modulus of the error in the mid-range of the interval. The authors call the former the (E-E) case and the latter the (E-M) case.

In this chapter the authors describe methods for getting a more uniform approximation throughout the interval of approximation in the two aforementioned cases (E-E) and (E-M). They also discuss improving approximation of the derivative obtained by differentiating the constructed interpolation approximations.

In Chapter 3, H. R. Fernández-Morales, A. G. Garcia, and G. Pérez-Villalón consider sampling in a general shift-invariant space $V_2(\phi)$ of $L^2(\mathbb{R}^d)$ with a set Φ of r stable generators and in which the data are samples of some filtered versions of the signal itself taken at a sub-lattice of $\mathbb{Z}^d$. The authors call this problem the problem of generalized sampling in shift-invariant spaces. Assuming that the ℓ^2-norm of the generalized samples of any $f \in V_2(\phi)$ is stable with respect to the $L^2(\mathbb{R}^d)$-norm of the signal f, the authors derive frame expansions in the shift-invariant subspace $V_2(\phi)$ allowing the recovery of signals in this space from the available data.

A similar sampling problem is considered by M. Nashed and Q. Sun in Chapter 4, where they consider a variety of Hilbert and Banach spaces of functions that admit sampling expansions of the form $f(t) = \sum_{n=1}^{\infty} f(t_n) S_n(t)$, where $\{S_n(t)\}_{n=1}^{\infty}$ is a family of functions that depend on the sampling points $\{t_n\}$ but not on the function f. Those function spaces, which arise in connection with sampling expansions, include reproducing-kernel spaces, Sobolev spaces, shift-invariant spaces, translation-invariant spaces and spaces of signals with finite rate of innovation. The authors first discuss the engineering approach to the Shannon sampling theorem which is based on trains of delta functions and then try to provide rigorous justification to the engineering approach using distribution theory and generalized functions. They also discuss sampling in some reproducing-kernel Banach spaces.

Chapter 5 by P. Vaidyanathan and P. Pal gives an overview of the concept of coprime sampling and its applications. Coprime sampling was recently introduced by the authors first for the case of one-dimensional signals and then extended to multidimensional signals. The basic idea is that a continuous-time (or spatial) signal is sampled simultaneously by two sets of samplers, with sampling rates $1/NT$ and $1/MT$ where M and N are coprime integers and $T > 0$. One of the main results is that it is possible to estimate the autocorrelation of the signal at a much higher rate $1/T$ than the total sampling rate. Thus, any application which is based on autocorrelation will benefit from such sampling and reconstruction. An interesting mathematical problem that comes up when coprime sampling is extended to higher dimensions is how to generate a pair of integer matrices M and N which are commuting and coprime.

Chromatic derivatives and series expansions have recently been introduced as an alternative representation to Taylor series for bandlimited functions and they have been shown to be more useful in practical applications than Taylor series. Chromatic series have similar properties to those of the Whittaker–Shannon–Kotel'nikov sampling series. In Chapter 6, A. Zayed gives an overview of chromatic derivatives and series in one and several variables and then use the Bargmann transform to show that functions, in the Bargmann–Segal–Foch space $\mathfrak{F}$, which is a reproducing-kernel Hilbert Space of entire functions, can be represented by chromatic series expansions. As a result, some properties of the Bargmann–Segal–Foch space can be deduced from those of chromatic series.

In Chapter 7, by D. Alpay, P. Jorgensen, I. Lewkowicz, and I. Marziano, the authors use functional analysis techniques to solve interpolation problems not in the context of bandlimited functions or shift-invariant spaces but in the setting of the Hardy space $H^2(\mathbf{D})$ of functions analytic in the open unit disk $\mathbf{D}$. The space $H^2(\mathbf{D})$ plays an important role in complex analysis, signal processing, and linear dynamical systems. Recently the Cuntz semigroups of C^*-algebras and the Cuntz relations for positive elements in a C^*-algebra have attracted some attention because of their newly discovered connections with applications in signal processing, sub-band filters, and wavelets, which all fall within a larger context of multiscale analysis.

In this work the authors study the Cuntz relations in a different context. They introduce connections between the Cuntz relations and the Hardy space $H^2(\mathbf{D})$ and then use a decomposition of elements in $H^2(\mathbf{D})$ associated with certain isometries which satisfy the Cuntz relations, to solve a new kind of multipoint interpolation problem in $H^2(\mathbf{D})$ where for instance only a linear combination of the values of a function at given points is preassigned, rather than the values at the points themselves.

Chapter 8 by J. Benedetto and S. Datta deals with the autocorrelation of sequences and the construction of constant amplitude zero autocorrelation (CAZAC) sequences x on the integers $\mathbb{Z}$ by means of Hadamard matrices. Recall that a real Hadamard matrix is a square matrix whose entries are either $+1$ or -1 and whose rows are mutually orthogonal. First, the authors explain from a practical point of view why constant amplitude and zero autocorrelation sequences are important. The zero autocorrelation property ensures minimum interference between signals sharing the same channel.

The authors review properties and problems related to Hadamard matrices and then establish the relation between CAZAC sequences on $\mathbb{Z}/N\mathbb{Z}$, Hadamard matrices, and the discrete Fourier transform. They proceed to construct CAZAC sequences on $\mathbb{Z}$ by means of Hadamard matrices and construct unimodular functions on $\mathbb{Z}$ whose autocorrelations are triangles.

Part II, Multiscale Analysis, consists of Chapters 9–12. The authors in Chapter 9 explain that chaos and random fractal theories, which have been used in the analysis of complex data, are fundamentally two different theories. Chaos theory shows that irregular behaviors in a complex system may be generated by nonlinear deterministic processes, while noise or randomness does not play any role. On the other hand, random fractal theory assumes that the dynamics of the system are inherently random.

Since the two theories are different, different conclusions may be drawn depending upon which theory is used to analyze the data. A great deal of research has been devoted to determining whether a complex time series is generated by a chaotic or a random system. The authors discuss the scale-dependent Lyapunov exponent (SDLE) and use it to develop a unified multiscale analysis theory of complex data.

The goal of Chapter 10, by E. Lin, M. Haske, M. Smith, and D. Sowards, is to determine the optimal wavelet, order, level, and threshold for denoising and compressing an ECG (electrocardiogram) signal while smoothing out and maintaining the integrity of the original signal. The wavelets used are: Daubechies, Biorthogonal Spline, Coiflet, and Symlet. Various thresholds have been utilized, such as soft, hard, global, rigorous SURE, heuristic SURE, universal, and minimax. But, the two kinds of thresholding that are used extensively in this chapter are hard and soft thresholding.

It is well known that the Hermite functions are an orthogonal basis for $L^2(\mathbb{R})$. They are also eigenfunctions of a Sturm–Liouville differential operator, as well as the Fourier transform. D. Mugler and A. Mahadevan in Chapter 11 call these functions the continuous Hermite functions (CHF) to distinguish them from another set of functions that they introduced in a previous work and called the discrete Hermite functions (DHF).

The DHF have the analogous property that they are eigenvectors of a shifted (centered) Fourier matrix and they also form an orthonormal set in a vector space. The authors discuss the notion of Gaussian derivatives and their relationship to the Hermite functions. Because of their relationship with the Gaussian derivatives, the CHF have been used for the multiscale Hermite transform. Multiscale analysis in this chapter refers to the ability to zoom in on features in a signal, moving from a coarse approximation to include details at several different levels. In particular, multiscale analysis provides a decomposition of the input signal into an approximation signal and detail signals at several different levels. The main goal of this chapter is to extend these results to the discrete case. The discrete Hermite transform analysis of an input signal is then compared to the wavelet analysis of the same signal at three different levels.

Local discriminant basis (LDB), which was developed by Saito and Coifman for the purpose of classification, is a tool to extract useful features in signal and image classification problems. It works by decomposing training signals into a time–frequency dictionary. The dictionaries decompose signals into a redundant set of orthogonal subspaces. Each subspace contains basis vectors localized in time and frequency. The goal is, given a dictionary, to find the signal representation within the dictionary that is most useful for classification.

In Chapter 12, B. Marchand and N. Saito propose the use of signatures and earth mover's distance (EMD) to provide data adaptive statistic that is more descriptive than the distribution of energies and more robust than an epdf (empirical probability density function)-based approach.

The authors first review LDB and EMD and then outline how they can be incorporated into a fast EMD-based LDB algorithm and then compare its performance with different LDB algorithms. They also demonstrate the capabilities of their new algorithm, in comparison with both energy distribution and epdf-based LDB algorithms, by using four different classification problems made of synthetic data sets.

Part III, Statistical Analysis, is comprised of Chapters 13–15. The problem of characterizing a probability distribution is an important problem in various fields. Various systems of distributions have been constructed to provide approximations to a wide variety of distributions. These systems are designed with the requirements of computational ease and feasibility of algebraic manipulation.

In Chapter 13, G. Hamadani focuses on the characterization of the Amoroso distribution, which is a four-parameter, continuous, univariate, unimodel pdf, with semi-infinite range. Many well-known and important distributions are special cases or limiting cases of the Amoroso distribution. The author gives characterizations of the Amoroso distribution in two separate cases based on the truncated moment of a function of first-order statistic and of a function of nth order statistic. He also presents similar characterizations of other distributions, such as SSK, SKS, SK, and SKS-type distributions.

Bayesian paradigm is popular in wavelet data processing because Bayes rules are shrinkers. The Bayes rules can be constructed to mimic the thresholding rules for wavelets, i.e., to slightly shrink the large coefficients and heavily shrink the small coefficients. A paradigmatic task in which wavelets are typically applied is the recovery of an unknown signal f from noisy measurements.

In Chapter 14 by N.Reményi and B. Vidakovic, the authors review some of these concepts and discuss different Bayesian wavelet regression models and methods for wavelet shrinkage. As an illustration of the Bayesian approach, they present BAMS (Bayesian adaptive multiresolution shrinkage) method.

The subject of Chapter 15, the last chapter of the monograph, is the so-called statistical learning theory. One of the central problems in the statistical learning theory is this: given some empirical data $Z = \{(x_i, y_i), i = 1, 2, \ldots n\}$, construct the estimator $f : X \to Y$ that approximates best the relationship between the input x and the output y of a system, i.e., $y \approx f(x)$. The data are seen as the realizations of random variables $(x, y) \in X \times Y$ with a probability density $p(x, y)$. The theory suggests an approach for constructing an estimator that is based on an operator equation for the estimator. The authors, S. Lu, S. Pereverzyev Jr, and S. Sampath, discuss this operator equation and show how it can be treated by the recently developed multiparameter regularization methods, the dual regularized total least squares (DRTLS) and the multi-penalty regularization (MPR).

Finally, the editors would like to express their gratitude to Professor Gilbert Walter for his support and guidance over the years and more importantly for his kindness and friendship. We also wish to thank the Springer-Verlag editors for their support, in particular, Mr. Steven Elliot, who initiated the project, and Merry Stubber and Dr. Alison Waldron for their help throughout the project. The first editor would like to thank the Battlefield Visualization Branch at the US Air Force Research Lab (AFRL) and the Ohio University Faculty Fellowship Leave program for their generous financial support. Her particular thanks are given to Dr. Jeffery Connor (Ohio University), Dr. Paul Havigs (AFRL), and Ms. Kathryn Farris (AFRL) for their encouragement and support.

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