CS de señales analógicas

Miguel A. Hernández-Medina

[1]Departamento de Matemática Aplicada
a las Tecnologías de la Información
ETSIT, Universidad Politécnica de Madrid

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Parte I

Demodulador Aleatorio

Random Demodulator

Modelo

$$\boxed{f(t) = \sum_{w \in \Omega} a_\omega e^{-2\pi i \omega t}, \quad t \in [0, 1)}$$

$$\Omega \subset \{0, \pm 1, \pm 2, \ldots, \pm(W/2 - 1), W/2\},\ |\Omega| = K,\ K \ll W$$

- Bandalimitadas
- Periódicas
- Dispersas

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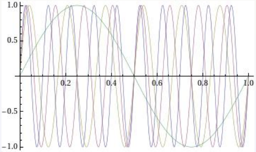

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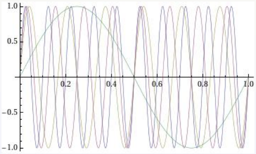

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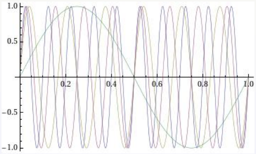
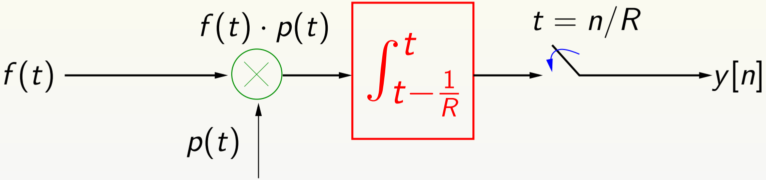


Demodulador Aleatorio

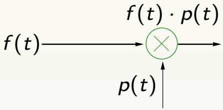
Sistema



$$R \ll W$$

Demodulador Aleatorio

Señal de demodulación



- $\varepsilon_0, \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{W-1}$ toman los valores ± 1 con igual probabilidad

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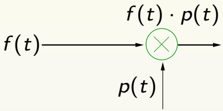
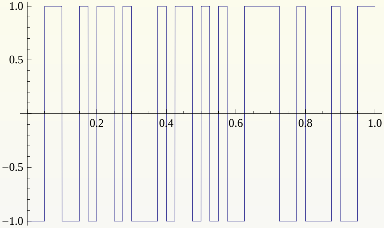




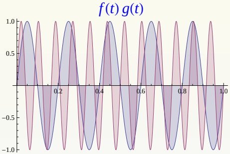
- $\varepsilon_0, \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{W-1}$ toman los valores ± 1 con igual probabilidad

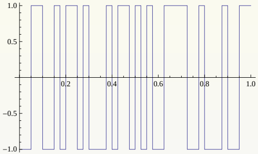
$$p(t) = \varepsilon_n, \quad t \in \left[\frac{n}{W}, \frac{n+1}{W}\right), \quad n = 0, \ldots W-1$$

- p cambia entre los niveles ± 1 a una tasa de W Hz.

Demodulador Aleatorio

Análisis en Frecuencia





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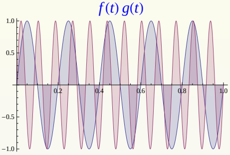

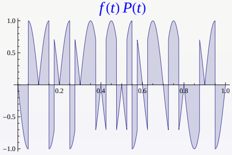

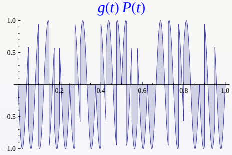

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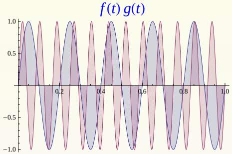
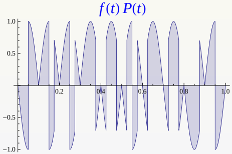
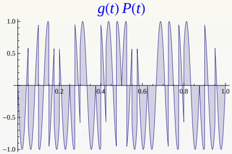
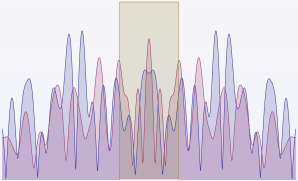


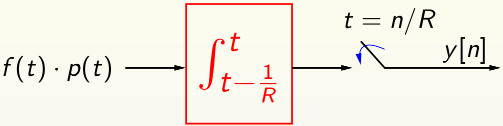






Demodulador Aleatorio

Filtro paso bajo

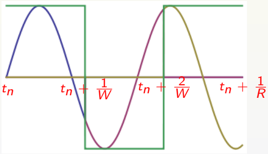


$\boxed{W = RS}$ $\boxed{R \ll W}$

$$y[m] = R \int_{\frac{m}{R}}^{\frac{m+1}{R}} f(t)p(t)\, dt \quad m = 0, 1, \ldots, R-1$$

Vector de amplitud-fase

Supongamos que $W = 3R$ y sea $t_n = \dfrac{n}{W}$



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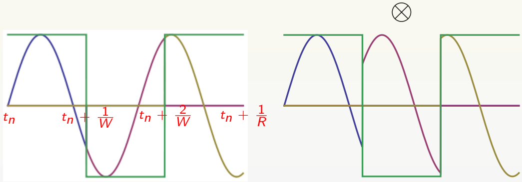

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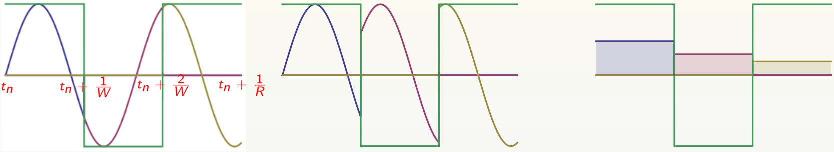
$$\otimes \qquad R(x_n - x_{n+1} + x_{n+3}) = y_n$$



$$\boxed{x_n = \int_{t_n}^{t_n + \frac{1}{W}} f(t)\,dt,}$$

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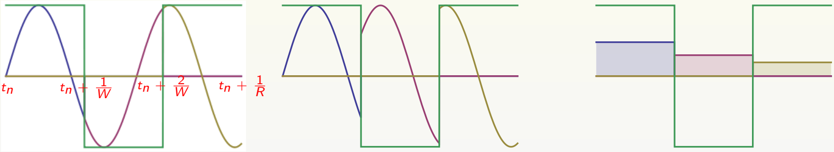

$$\boxed{x_n = \int_{t_n}^{t_n + \frac{1}{W}} f(t)dt, \quad y_n = R \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}$$

Vector de amplitud-fase

$$\boxed{f(t) = \sum_{w \in \Omega} a_\omega e^{-2\pi i \omega t}}$$

$$x_n = \int_{t_n}^{t_n + \frac{1}{W}} f(t)\, dt = \sum_{\omega \in \Omega} a_\omega \left[\frac{e^{-2\pi i \omega / W} - 1}{2\pi i \omega} \right] e^{-2\pi i \omega n / W}$$

Vector de amplitud-fase

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$$\boxed{s_\omega = a_\omega \left[\frac{e^{-2\pi i\omega/W}-1}{2\pi i\omega}\right]}$$

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$$\mathbf{s} = [\mathbf{s}[0], \mathbf{s}[1], \mathbf{s}[-1], \ldots, \mathbf{s}[W/2]]$$
$$\mathbf{s}[\omega] = \sqrt{W} s_\omega,\ \ \omega \in \Omega; \quad \mathbf{s}[\omega] = 0,\ \ \omega \notin \Omega$$

s  *K*-disperso

Vector de amplitud-fase

Expresión matricial

$$< \mathbf{F} >_{n,\omega} = \frac{1}{\sqrt{W}} e^{-2\pi i \omega n / W},$$

$$n = 0, 1, \ldots, W - 1; \ \omega = 0, \pm 1, \ldots, \pm(\frac{W}{2} - 1), \frac{W}{2}$$

- **F** es una permutación de la matriz de Fourier de orden W
- **F** es ortogonal.

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$$\boxed{\mathbf{x} = \mathbf{F}\mathbf{s}}$$

Expresión matricial del demodulador

- Sea $\varepsilon_0, \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{W-1}$ la sucesión que define p $(\varepsilon_n = \pm 1)$

$$\mathbf{D} = \begin{bmatrix} \varepsilon_0 & & & \\ & \varepsilon_1 & & \\ & & \ddots & \\ & & & \varepsilon_{W-1} \end{bmatrix}$$

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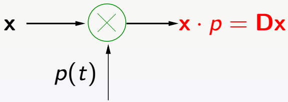


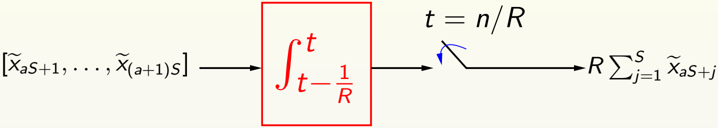
$$\boxed{\widetilde{\mathbf{x}} = \mathbf{D}\mathbf{x}}$$

Expresión matricial del filtro

- $W = RS$

$$\mathbf{Dx} = \widetilde{\mathbf{x}} = \left[\widetilde{x}_1, \cdots, \widetilde{x}_S, \widetilde{x}_{S+1}, \cdots, \widetilde{x}_{2S}, \cdots \widetilde{x}_{(R-1)S+1}, \cdots, \widetilde{x}_{RS}\right]^\top$$

Expresión matricial del filtro



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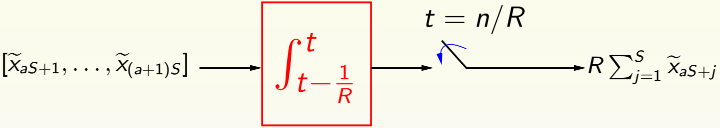

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$$\mathbf{H}\mathbf{D}\mathbf{x} = \mathbf{y}$$

$$\mathbf{H} = R\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & & & & & \\ & & & & & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad (R = 2, S = 5)$$

Random Demodulator

Matriz de demodulación aleatoria

$$\boxed{\mathbf{HDFs} = \mathbf{y}}$$

- $\mathbf{s}$ es un vector K-disperso
- $\Phi = \mathbf{HDF}$ es la matriz de demodulación aleatoria

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¿Es Φ una matrix CS?

Recuperación de la señal

- $\Phi =$ **HDF** matriz de demodulación aleatoria
- $\mathbf{s} \in \mathbb{C}^W$ K-disperso

$$\widehat{\mathbf{s}} = \mathrm{argmin}_{\mathbf{z} \in \mathbb{C}^N} \|\mathbf{z}\|_1 \text{ tal que } \Phi\mathbf{z} = \Phi\mathbf{s}$$

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Si $R \approx 1{,}7K \log(W/K + 1)$

$$\widehat{\mathbf{s}} = \mathbf{s} \quad \text{con probabilidad} > 0{,}99$$

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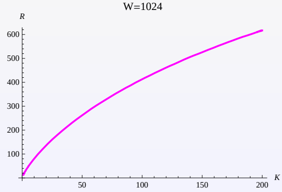
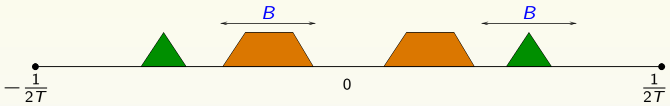


Parte II

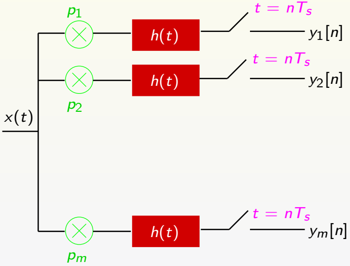
Señales multibanda dispersas

Modelo



- $x(t)$ señal real,
- El soporte de $X(\omega)$ está contenido en $[-\frac{1}{2T}, \frac{1}{2T}]$
- El contenido en frecuencia está contenido en N ($N = 2K$) bandas disjuntas de anchura a lo más B,
- $NB \ll \dfrac{1}{T}$

Sistema



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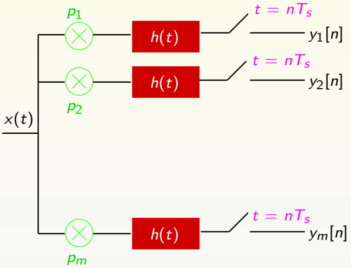
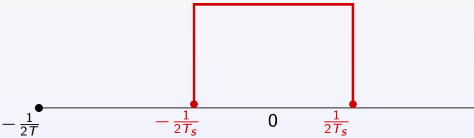


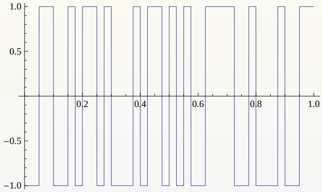
- $h(t)$ filtro ideal con frecuencia de corte $1/(2T_s)$



Demodulador aleatorio

- p_i es T_p-periódica
- $\varepsilon_{i0}, \varepsilon_{i1}, \ldots, \varepsilon_{i(M-1)}$ toman los valores ± 1 con igual probabilidad

$$p_i(t) = \varepsilon_{ik}, \quad t \in \left[k\frac{T_p}{M}, (k+1)\frac{T_p}{M} \right)$$



Parámetros del sistema

f	frecuencia Nyquist
m	número de canales
T_p	periodo de p_i
$1/(2T_s)$	frecuencia de corte del filtro paso bajo
M	número de ± 1 intervalos en cada periodo de p_i

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Suposiciones sobre los parámetros

- Si $f_s = T_s^{-1}$ y $f_p = T_p^{-1}$
  - $f_s \geq f_p \geq B$
- $m \geq 2N$
- $M \geq 2\left\lceil \frac{f}{2f_p} + \frac{1}{2} \right\rceil - 1$

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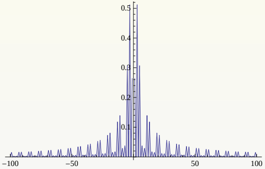
Análisis en frecuencia

- Serie de Fourier de p_j

$$p_j(t) = \sum_{l=-\infty}^{\infty} c_{jl} e^{-2\pi i l t / T_p}$$



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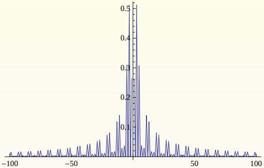
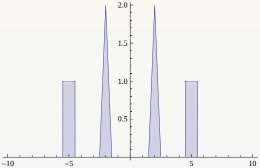


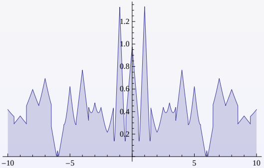
- Tranf. de Fourier de $\widetilde{x}_j(t) = x_j(t) p_j(t)$

$$\widetilde{X}_j(\omega) = \int_{-\infty}^{\infty} \widetilde{x}_j(t) e^{-2\pi i \omega t}\, dt =$$

$$= \sum_{l=-\infty}^{\infty} c_{jl} X(\omega - l f_p)$$

Para cada ω, la suma tiene a lo más $\lceil f / f_p \rceil$ términos no nulos.

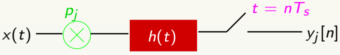




Análisis en frecuencia

- DTFT de $y_j[n]$

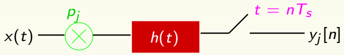
$$Y_j(e^{2\pi i\omega T_s}) = \widetilde{X}_j(\omega)\cdot H(\omega) =$$
$$= \sum_{l=-L_0}^{L_0} c_{jl} X(\omega - l f_p)$$



  - $\omega \in [-f_s/2, f_s/2] = \mathcal{F}_s$
  - $L_0 = \left\lceil \dfrac{f + f_s}{2f_p} \right\rceil - 1$

Análisis en frecuencia

- DTFT de $y_j[n]$



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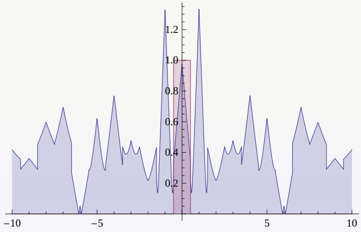

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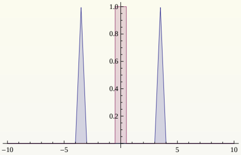
Problema CS

$$Y_j(e^{2\pi i\omega T_s}) = \sum_{n=-L_0}^{L_0} c_{jl} X(\omega - l f_p), \quad \omega \in \mathcal{F}_s$$

$$\mathbf{Y}(\omega) = \mathbf{A}\mathbf{Z}(\omega), \quad \omega \in \mathcal{F}_s$$

- $\mathbf{Y}_j(\omega) = Y_j(e^{2\pi i\omega T_s})\ j = 1, 2, \ldots, m,\ \omega \in \mathcal{F}_s$
- $\mathbf{Z}_k(\omega) = X(\omega + (k - L_0 - 1) f_p)\ k = 1, 2, \ldots, L = 2L_0 + 1,\ \omega \in \mathcal{F}_s$
- $\mathbf{A} \in \mathbb{C}^{m \times L}\ \langle \mathbf{A} \rangle_{jl} = c_{i-l} = c_{jl}^*$

El vector $\mathbf{Z}(\omega)$. Dispersión uniforme



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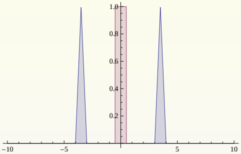

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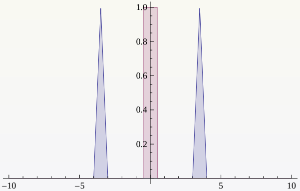

$\mathbf{z}$ | | | | | | | | | | |

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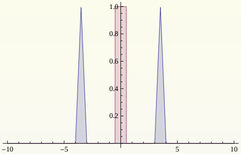

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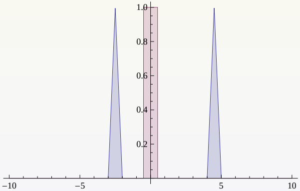

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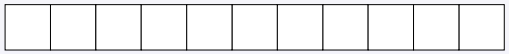

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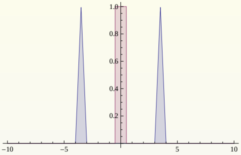

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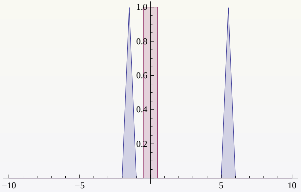

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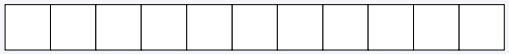
Z 

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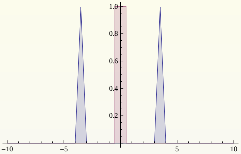

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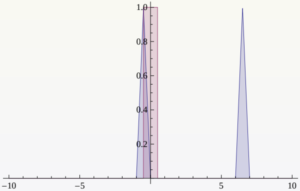

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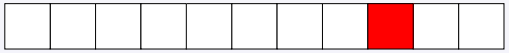
Z 

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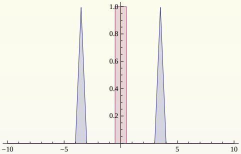

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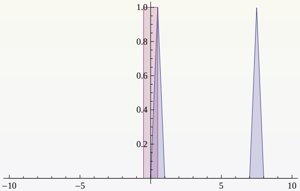
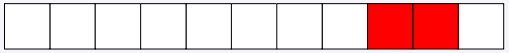
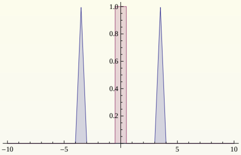


z 

El vector $\mathbf{Z}(\omega)$. Dispersión uniforme



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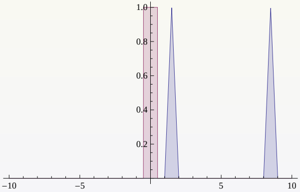
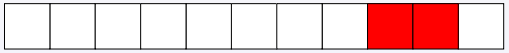


$\mathbf{Z}$



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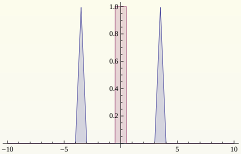

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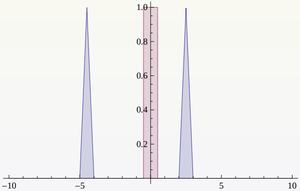

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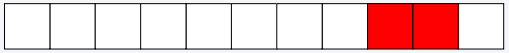

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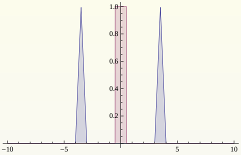

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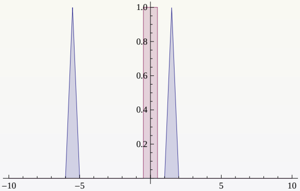

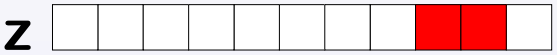

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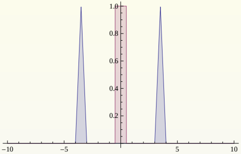

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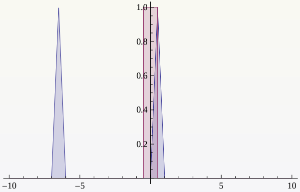

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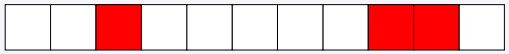
z 

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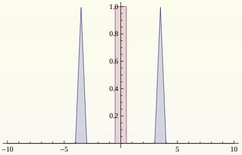

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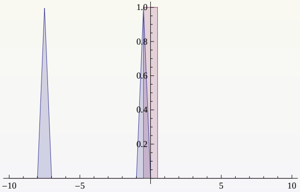

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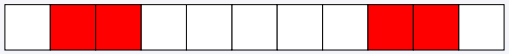
$\mathbf{Z}$ 

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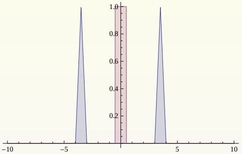

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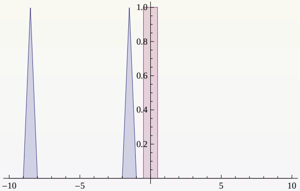
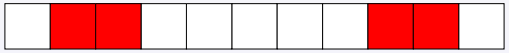


z 

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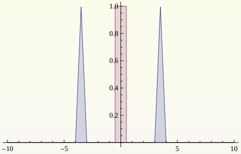

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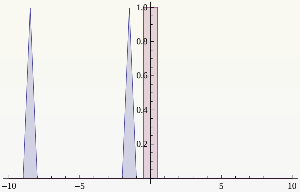

Z 

$$\boxed{S = \{2, 3, 9, 10\}}$$

A es una matriz CS

Supondremos que para cada $\omega \in \mathcal{F}_s$

$$\mathbf{Y}(\omega) = \mathbf{A}\mathbf{Z}(\omega)$$

tiene una solución $\overline{\mathbf{Y}}(\omega)$ $|S|$-dispersa

Solución cuando el soporte es conocido

- $S = \mathrm{supp}(\mathbf{Z}(\omega))$ **(Conocido)**
- $\mathbf{A}_S$ submatriz de $\mathbf{A}$ con las columnas indicadas por S
- $\mathbf{A}_S$ tiene rango completo

$$\mathbf{A}_S^\dagger = (\mathbf{A}_S^H \mathbf{A}_S)^{-1} \mathbf{A}_S^H$$

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Solución

$$\mathbf{Z}_S(\omega) = \mathbf{A}_S^{\dagger} \mathbf{Y}(\omega)$$
$$\mathbf{Z}_j(\omega) = 0, \ j \notin S$$

Cálculo del soporte

Se define la matriz

$$\mathbf{Q} = \int_{\omega \in \mathcal{F}_s} \mathbf{Y}(\omega)\mathbf{Y}^H(\omega) d\omega = \sum_{-\infty}^{+\infty} \mathbf{y}[n]\mathbf{y}^\top[n] \in \mathbb{C}^{m \times m}$$

$$\mathbf{Q} = \mathbf{V}\mathbf{V}^H$$

$\mathbf{V}$ es un frame de $\mathbf{Y}(\omega)$

Las columnas de $\mathbf{V}$ generan span$\{\mathbf{Y}(\omega)\}$ para cada $\omega \in \mathcal{F}_s$

Cálculo del soporte

La ecuación matricial

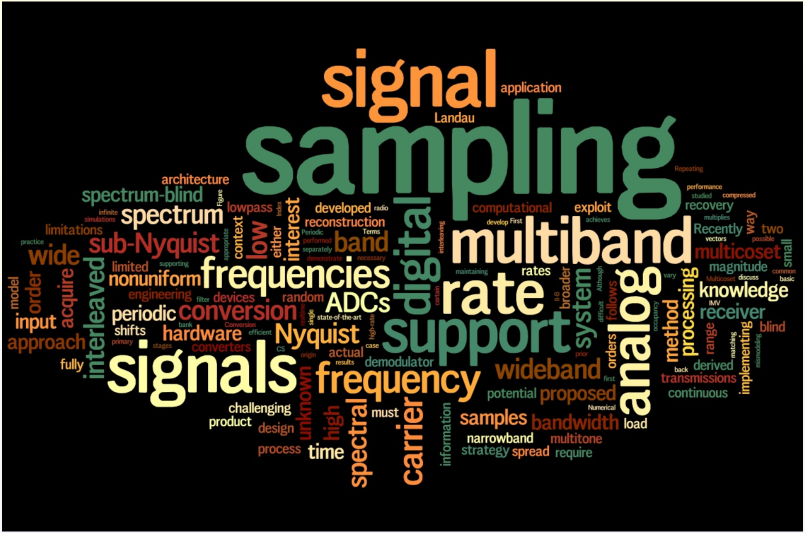
$$\mathbf{V} = \mathbf{A}\mathbf{U}$$

tiene una única solución $|S|$-dispersa $\overline{\mathbf{U}}$ y

$$I(\overline{\mathbf{U}}) = \{j \mid \text{ fila } j \text{ de } \overline{\mathbf{U}} \neq 0\} = S$$

Conclusiones

- El CS de señales analógicas es aún muy limitado
- El potencial de estudio e investigación en este campo son muy grandes
  1. Espacios invariantes por traslación,
  2. Utilización de otras transformadas o representaciones
  3. etc.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system
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narrowband
multitone
process
time
strategy
spread
require
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carrier
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interest
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context
model
order
acquire
orders
follows
range
implementing
broader
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small

¿Preguntas?