Structured eigenvalue condition numbers for parameterized quasiseparable matrices

Froilán M. Dopico

Department of Mathematics and ICMAT, Universidad Carlos III de Madrid, Spain

18th Conference of the International Linear Algebra Society
June 3-7, 2013, Providence, RI, USA

Abstract

- Fast computations with $n \times n$ **rank structured matrices** have received recently a lot of attention and they rely on **parameterizing or representing** these matrices in terms of $O(n)$ parameters.
- If **parameters are the input of an algorithm for computing** Y, then the variation of Y under tiny relative **perturbations** of the parameters **determines the maximal/ideal accuracy of a computation**, and,
- in addition, this allows us to decide rigourously **whether or not one representation is better than another for computing** Y accurately.
- In this talk, we consider
  1. $Y = \lambda$, a simple eigenvalue.
  2. Mainly, **{1,1}-quasiseparable** matrices.
  3. A few comments on **{n_L,n_U}-quasiseparable** matrices.
  4. **Two representations**: **Quasiseparable** (Eidelman-Gohberg) and **Givens-vector** (Vandebril-Van Barel-Mastronardi).
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Main characters: The matrix, the eigenvalue, the eigenvectors,...

Let $\lambda \in \mathbb{C}$ be a **simple eigenvalue** of $M \in \mathbb{R}^{n\times n}$, with left and right eigenvectors $\boldsymbol{y} \in \mathbb{C}^n$ and $\boldsymbol{x} \in \mathbb{C}^n$, that is,

$$M\,\boldsymbol{x} = \boldsymbol{x}\,\lambda \qquad \text{and} \qquad \boldsymbol{y}^*\,M = \lambda\,\boldsymbol{y}^*.$$

Theorem (The Wilkinson Condition Number (Wilkinson, 1965))

$$\kappa_\lambda := \lim_{\eta\to 0} \sup \left\{ \frac{|\delta\lambda|}{\eta} : (\lambda+\delta\lambda) \textit{ is eigenvalue of } (M+\delta M),\ \|\delta M\|_2 \le \eta \right\}$$
$$= \frac{\|\boldsymbol{y}\|_2 \|\boldsymbol{x}\|_2}{|\boldsymbol{y}^*\boldsymbol{x}|}$$

Wilkinson condition number is an **absolute-absolute normwise** condition number, so, it is **not convenient** in most applications.

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Theorem (Relative Wilkinson Condition Number)

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$$= \frac{\|y\|_2 \|x\|_2}{|y^* x|} \frac{\|M\|_2}{|\lambda|} \qquad \text{(relative-relative normwise)}$$

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Note that $\mathrm{cond}(\lambda) \leq \sqrt{n}\, \kappa_\lambda^{rel}$ and, in some important situations, $\ll$

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Theorem (relative-relative componentwise for parameters)

Let $M \in \mathbb{R}^{n\times n}$ be a matrix whose entries are differentiable functions of a set of parameters $\Omega = (\omega_1, \omega_2, \ldots, \omega_N) \in \mathbb{R}^N$. This is denoted as $M(\Omega)$. Let λ be a simple eigenvalue of $M(\Omega)$ with left/right eigenvectors $\boldsymbol{y}$, $\boldsymbol{x}$ and define

$$\mathrm{cond}(\lambda;\Omega) := \lim_{\eta\to 0} \sup \left\{ \frac{|\delta\lambda|}{\eta|\lambda|} : (\lambda+\delta\lambda) \text{ e-value of } M(\Omega+\delta\Omega),\ |\delta\Omega| \le \eta|\Omega| \right\}.$$

Then

$$\mathrm{cond}(\lambda;\Omega) = \frac{1}{|\lambda|\,|\boldsymbol{y}^*\boldsymbol{x}|} \sum_{i=1}^{N} \left| \boldsymbol{y}^* \left(\omega_i \frac{\partial M}{\partial \omega_i} \right) \boldsymbol{x} \right|$$

Sometimes, it is necessary to specify both the matrix and the parameters:

$$\mathrm{cond}(\lambda, M; \Omega) \equiv \mathrm{cond}(\lambda;\Omega)$$

Structured cond. nums. for parameterized structured matrices

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Definition (Eidelman-Gohberg, Int. Eq. Op. Th., 1999)

A square matrix $C \in \mathbb{R}^{n \times n}$ is a $\{n_L, n_U\}$**-quasiseparable** matrix if

- every submatrix of C entirely located in the **strictly lower (resp. upper) triangular part** of C **have rank at most** n_L (resp. n_U), and
- at least one of these submatrices has rank equal to n_L (resp. n_U).

This is equivalent to

$$\max_i \operatorname{rank} C(i+1:n, 1:i) = n_L \quad \text{and} \quad \max_i \operatorname{rank} C(1:i, i+1:n) = n_U$$

Therefore the following submatrices have rank at most n_L or rank at most n_U:

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Quasiseparable matrices (II)

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Quasiseparable matrices (II): rank $\leq n_L$

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Theorem (Eidelman and Gohberg, Int. Eq. Op. Th., 1999)

The set of $\{1,1\}$**-quasiseparable matrices of size** $n \times n$ **can be parameterized in terms of** $7n-8$ **independent scalar parameters**

$$\Omega_{QS} = (\{p_i\}_{i=2}^{n}, \{a_i\}_{i=2}^{n-1}, \{q_i\}_{i=1}^{n-1}, \{d_i\}_{i=1}^{n}, \{g_i\}_{i=1}^{n-1}, \{b_i\}_{i=2}^{n-1}, \{h_i\}_{i=2}^{n})$$

as follows: C *is* $\{1,1\}$*-quasiseparable if and only if (*5×5 *example)*

$$C = \begin{bmatrix} d_1 & g_1h_2 & g_1b_2h_3 & g_1b_2b_3h_4 & g_1b_2b_3b_4h_5 \\ p_2q_1 & d_2 & g_2h_3 & g_2b_3h_4 & g_2b_3b_4h_5 \\ p_3a_2q_1 & p_3q_2 & d_3 & g_3h_4 & g_3b_4h_5 \\ p_4a_3a_2q_1 & p_4a_3q_2 & p_4q_3 & d_4 & g_4h_5 \\ p_5a_4a_3a_2q_1 & p_5a_4a_3q_2 & p_5a_4q_3 & p_5q_4 & d_5 \end{bmatrix}$$

This is the most general representation of $\{1,1\}$-quasiseparable matrices and includes as special cases other representations explained in the books by Vandebril, Van Barel, and Mastronardi (2008).

Quasiseparable representation of $\{1,1\}$-quasiseparable matrices

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Theorem (Eidelman and Gohberg, Int. Eq. Op. Th., 1999)

The set of $\{1,1\}$**-quasiseparable matrices of size** $n \times n$ **can be parameterized in terms of** $7n-8$ **independent scalar parameters**

$$\Omega_{QS} = (\,\{p_i\}_{i=2}^{n}, \{a_i\}_{i=2}^{n-1}, \{q_i\}_{i=1}^{n-1}, \{d_i\}_{i=1}^{n}, \{g_i\}_{i=1}^{n-1}, \{b_i\}_{i=2}^{n-1}, \{h_i\}_{i=2}^{n}\,)$$

as follows: C *is* $\{1,1\}$*-quasiseparable if and only if (*5×5 *example)*

$$C = \begin{bmatrix} d_1 & g_1h_2 & g_1b_2h_3 & g_1b_2b_3h_4 & g_1b_2b_3b_4h_5 \\ p_2q_1 & d_2 & g_2h_3 & g_2b_3h_4 & g_2b_3b_4h_5 \\ p_3a_2q_1 & p_3q_2 & d_3 & g_3h_4 & g_3b_4h_5 \\ p_4a_3a_2q_1 & p_4a_3q_2 & p_4q_3 & d_4 & g_4h_5 \\ p_5a_4a_3a_2q_1 & p_5a_4a_3q_2 & p_5a_4q_3 & p_5q_4 & d_5 \end{bmatrix}$$

This is the most general representation of $\{1,1\}$-quasiseparable matrices and includes as special cases other representations explained in the books by Vandebril, Van Barel, and Mastronardi (2008).

Componentwise eig. cond. number in the quasiseparable repr.

Theorem

Let $C \in \mathbb{R}^{n \times n}$ *be a* $\{1,1\}$*-quasiseparable matrix and*

$$C = C_L + C_D + C_U$$

with C_L *strictly lower triangular,* C_D *diagonal, and* C_U *strictly upper triangular. Then*

$$\begin{aligned}\mathrm{cond}(\lambda; \Omega_{QS}) = & \frac{1}{|\lambda||\boldsymbol{y}^* \boldsymbol{x}|} \Bigg\{ |\boldsymbol{y}^*||C_D||\boldsymbol{x}| + |\boldsymbol{y}^*||C_L \boldsymbol{x}| + |\boldsymbol{y}^* C_L||\boldsymbol{x}| \\ & + |\boldsymbol{y}^*||C_U \boldsymbol{x}| + |\boldsymbol{y}^* C_U||\boldsymbol{x}| \\ & + \sum_{i=2}^{n-1} \left| \boldsymbol{y}^* \begin{bmatrix} 0 & 0 \\ C(i+1:n, 1:i-1) & 0 \end{bmatrix} \boldsymbol{x} \right| \\ & + \sum_{j=2}^{n-1} \left| \boldsymbol{y}^* \begin{bmatrix} 0 & C(1:j-1, j+1:n) \\ 0 & 0 \end{bmatrix} \boldsymbol{x} \right| \Bigg\}\end{aligned}$$

Properties of $\mathrm{cond}(\lambda; \Omega_{QS})$

Property 1

$\mathrm{cond}(\lambda; \Omega_{QS})$ **does not depend on the parameters**. It only depends on the matrix entries, the eigenvalue, and the left-right eigenvectors.

Therefore, for any two sets Ω_{QS} and Ω'_{QS} of quasiseparable parameters of the same matrix C

$$\mathrm{cond}(\lambda; \Omega_{QS}) = \mathrm{cond}(\lambda; \Omega'_{QS})$$

Property 2 (Comparison with unstructured condition number)

$$\mathrm{cond}(\lambda; \Omega_{QS}) \leq n\, \mathrm{cond}(\lambda) = n \frac{|\boldsymbol{y}^*|\,|C|\,|\boldsymbol{x}|}{|\lambda|\,|\boldsymbol{y}^*\boldsymbol{x}|},$$

and "potentially" $\mathrm{cond}(\lambda; \Omega_{QS}) \ll \mathrm{cond}(\lambda)$.

Property 3

Assume that λ, $\boldsymbol{x}$, $\boldsymbol{y}$, and Ω_{QS} are known. Then $\mathrm{cond}(\lambda; \Omega_{QS})$ can be computed in $42n - 57$ **flops**.

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Outline

Theorem (Vandebril-Van Barel-Mastronardi, Num. Lin. Alg. Appl., 2005)

The set of $\{1,1\}$**-quasiseparable matrices of size** $n \times n$ **can be parameterized in terms of**

- $\{c_i, s_i\}_{i=2}^{n-1}$ *pairs of cosines-sines,*
- $\{v_i\}_{i=1}^{n-1}, \{d_i\}_{i=1}^{n}, \{e_i\}_{i=1}^{n-1}$ *independent scalar parameters,*
- $\{r_i, t_i\}_{i=2}^{n-1}$ *pairs of cosines-sines,*

as follows: *C is $\{1,1\}$-quasiseparable if and only if (5×5 example)*

$$C = \begin{bmatrix} d_1 & e_1 r_2 & e_1 t_2 r_3 & e_1 t_2 t_3 r_4 & e_1 t_2 t_3 t_4 \\ c_2 v_1 & d_2 & e_2 r_3 & e_2 t_3 r_4 & e_2 t_3 t_4 \\ c_3 s_2 v_1 & c_3 v_2 & d_3 & e_3 r_4 & e_3 t_4 \\ c_4 s_3 s_2 v_1 & c_4 s_3 v_2 & c_4 v_3 & d_4 & e_4 \\ s_4 s_3 s_2 v_1 & s_4 s_3 v_2 & s_4 v_3 & v_4 & d_5 \end{bmatrix}$$

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Givens-vector representation of $\{1,1\}$-quasiseparable matrices (II)

- Givens-vector representation was introduced to improve the numerical stability in solving eigenproblems with respect other representations.
- **Givens-vector representation is a particular case of quasiseparable representation seen before, i.e., a particular choice of Ω_{QS},**

$$\Omega_{QS}^{GV} := (\underbrace{\{c_i, s_i\}_{i=2}^{n-1}}_{p_i,\,a_i}, \underbrace{\{v_i\}_{i=1}^{n-1}}_{q_i}, \underbrace{\{d_i\}_{i=1}^{n}}_{d_i}, \underbrace{\{e_i\}_{i=1}^{n-1}}_{g_i}, \underbrace{\{t_i, r_i\}_{i=2}^{n-1}}_{b_i,\,h_i})$$

with $p_n = h_n = 1$,

- therefore, one might think that it makes no sense to study again eigenvalue condition numbers since they are independent of the particular choice of Ω_{QS},
- **but the subtle point here is that independent componentwise perturbations of Ω_{QS}^{GV} destroy the pairs cosine-sine**, and
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- Givens-vector representation is considered by some authors as the "most stable" representation of quasiseparable matrices, but I am not aware of a formal proof of this fact.
- I pretend to give a first step in this direction.
- **Givens-vector representation is unique** by fixing $c_i \geq 0$ and $r_i \geq 0$.
- **Givens-vector representation is NOT a parametrization**, since the pairs of cosines-sines $\{c_i, s_i\}_{i=2}^{n-1}$ are not independent parameters. The same happens for $\{r_i, t_i\}_{i=2}^{n-1}$.
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The **“tangents”** $l_i, u_i \in \mathbb{R}$ allow us to write

$$\{c_i, s_i\}_{i=2}^{n-1} = \left\{ \frac{1}{\sqrt{1+l_i^2}}, \frac{l_i}{\sqrt{1+l_i^2}} \right\}_{i=2}^{n-1},$$

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and note that **tiny relative perturbations of l_i produce tiny relative perturbations of $\{c_i, s_i\}$**, same for u_i and $\{r_i, t_i\}$.

Therefore, we use the following parameters:

Definition (Givens-vector parameters via tangents)

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Properties of $\mathrm{cond}(\lambda;\Omega_{GV})$

Property 1

$\mathrm{cond}(\lambda;\Omega_{GV})$ **does depend on the parameters** $\{c_i, s_i\}$ **and** $\{r_i, t_i\}$, but these parameters are uniquely determined by the entries.

Theorem (Property 2)

Let $C \in \mathbb{R}^{n\times n}$ be a $\{1,1\}$-quasiseparable matrix, Ω_{GV} be the tangent-Givens-vector parameters of C, and Ω_{QS} be any set of quasiseparable parameters of C, then

$$\mathrm{cond}(\lambda, C;\Omega_{GV}) \leq \mathrm{cond}(\lambda, C;\Omega_{QS})$$

This **proves rigorously** that Givens-vector is the **"most stable"** quasisep represent. of $\{1,1\}$-quasisep matrices for eigen-computations.

Property 3

Assume that λ, x, y, and Ω_{GV} are known. Then $\mathrm{cond}(\lambda;\Omega_{GV})$ can be computed in $56\,n -$ constant **flops**.

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Outline

Numerical tests

- Thousands of random numerical tests have been performed in MATLAB,
- computing eigenvalues-vectors with `eig` command and then computing

$$\mathrm{cond}(\lambda), \qquad \mathrm{cond}(\lambda; \Omega_{QS}), \qquad \mathrm{cond}(\lambda; \Omega_{GV}).$$

- Most of the times $\mathrm{cond}(\lambda) \approx \mathrm{cond}(\lambda; \Omega_{QS}) \approx \mathrm{cond}(\lambda; \Omega_{GV})$,
- but there are $\{1,1\}$-quasiseparable matrices such that $\mathrm{cond}(\lambda) \gg \mathrm{cond}(\lambda; \Omega_{QS})$. We have found

$$\frac{\mathrm{cond}(\lambda)}{\mathrm{cond}(\lambda; \Omega_{QS})} = 5.02 \cdot 10^{12}$$

in a certain 300×300 matrix.

- However in all our tests $\quad \dfrac{\mathrm{cond}(\lambda; \Omega_{QS})}{\mathrm{cond}(\lambda; \Omega_{GV})} < 15.$
- This raises the question if $\quad 1 \leq \dfrac{\mathrm{cond}(\lambda; \Omega_{QS})}{\mathrm{cond}(\lambda; \Omega_{GV})} \leq \alpha(n)???$, with $\alpha(n)$ some moderately increasing function of n or a constant.

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$$\frac{\mathrm{cond}(\lambda)}{\mathrm{cond}(\lambda;\Omega_{QS})} = 5.02 \cdot 10^{12}$$

  in a certain 300×300 matrix.
- However in all our tests $\dfrac{\mathrm{cond}(\lambda;\Omega_{QS})}{\mathrm{cond}(\lambda;\Omega_{GV})} < 15.$
- This raises the question if $1 \leq \dfrac{\mathrm{cond}(\lambda;\Omega_{QS})}{\mathrm{cond}(\lambda;\Omega_{GV})} \leq \alpha(n)$???, with $\alpha(n)$ some moderately increasing function of n or a constant.

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- Thousands of random numerical tests have been performed in MATLAB,
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Outline

Quasiseparable representation of $\{n_L, n_U\}$-quasiseparable matrices

Theorem (Eidelman and Gohberg, Int. Eq. Op. Th., 1999)

The set of $\{n_L, n_U\}$**-quasiseparable matrices of size** $n \times n$ **can be parameterized in terms of parameters**

$$\Omega_{QS} = \left(\underset{1\times n_L}{\{p_i\}_{i=2}^{n}}, \underset{n_L\times n_L}{\{a_i\}_{i=2}^{n-1}}, \underset{n_L\times 1}{\{q_i\}_{i=1}^{n-1}}, \underset{1\times 1}{\{d_i\}_{i=1}^{n}}, \underset{1\times n_U}{\{g_i\}_{i=1}^{n-1}}, \underset{n_U\times n_U}{\{b_i\}_{i=2}^{n-1}}, \underset{n_U\times 1}{\{h_i\}_{i=2}^{n}} \right)$$

as follows: C is $\{n_L, n_U\}$-quasiseparable if and only if (5×5 example)

$$C(\Omega_{QS}) = \begin{bmatrix} d_1 & g_1h_2 & g_1b_2h_3 & g_1b_2b_3h_4 & g_1b_2b_3b_4h_5 \\ p_2q_1 & d_2 & g_2h_3 & g_2b_3h_4 & g_2b_3b_4h_5 \\ p_3a_2q_1 & p_3q_2 & d_3 & g_3h_4 & g_3b_4h_5 \\ p_4a_3a_2q_1 & p_4a_3q_2 & p_4q_3 & d_4 & g_4h_5 \\ p_5a_4a_3a_2q_1 & p_5a_4a_3q_2 & p_5a_4q_3 & p_5q_4 & d_5 \end{bmatrix}$$

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- The explicit expression of $\mathrm{cond}(\lambda; \Omega_{QS})$ is omitted, since it is somewhat messy although computable in $O((n_L^2 + n_U^2)n)$ flops.
- **Property 1:** $\mathrm{cond}(\lambda; \Omega_{QS})$ **depends on the particular choice of parameters** Ω_{QS} **and not only on the matrix entries.**
- **Property 2:** It may happen $\mathrm{cond}(\lambda; \Omega_{QS}) \gg \mathrm{cond}(\lambda)$, i.e., **unstructured smaller than structured!!**.

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Outline

1. Basics on eigenvalue condition numbers
2. Quasiseparable matrices
3. Condition numbers for $\{1,1\}$-quasiseparable matrices in the quasiseparable representation
4. Condition numbers for $\{1,1\}$-quasiseparable matrices in the Givens-vector representation
5. Numerical tests
6. Condition numbers for $\{n_L, n_U\}$-quasiseparable matrices in the quasiseparable representation
7. **Conclusions and future work**

Conclusions and future work

- We have presented a framework that allows us to obtain structured eigenvalue condition numbers for different representations of quasiseparable matrices.
- **For $\{1,1\}$-quasi. matrices the structure plays a key role and leads to eigenvalue condition numbers that can be much smaller, but in the $\{n_L, n_U\}$-case the selection of proper parameters is essential.**
- We still need to understand well the relationship between the eigenvalue condition numbers in the quasiseparable and Givens-vector representations for $\{1,1\}$-quasiseparable matrices,
- and to develop the Givens-Weight condition numbers for $\{n_L, n_U\}$-quasiseparable matrices.
- **Next step:** condition numbers for the solution of linear systems.

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