Low rank perturbation of canonical forms

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Outline

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DAEs and canonical forms

D(ifferential)**A**(lgebraic)**E**(quation):

$$A_0\, x(t) + A_1\, x'(t) = f(t) \qquad (1)$$

$A_0, A_1 \in \mathbb{C}^{m\times n}$, $x(t)$ unknown.

☞ Associated to the pencil: $A_0 + \lambda A_1$

☞ Canonical form of the pencil under (strict) equivalence:

$$E(A_0 + \lambda A_1)F = K_{A_0} + \lambda K_{A_1}$$

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Which canonical forms?

We are interested in canonical forms under Strict equivalence of regular matrix pencils: $E(A_0 + \lambda A_1)F$ (E, F nonsingular).

- $A_1 = -I$: Jordan canonical form (JCF) of A_0
- Weierstrass canonical form (WCF): regular pencils.

Low rank perturbations

$$A_0+\lambda A_1 \rightsquigarrow (A_0+\lambda A_1)+(B_0+\lambda B_1)=(A_0+B_0)+\lambda(A_1+B_1)$$

with $B_0+\lambda B_1$ of low rank (?????)

GOAL: Describe the generic change of the canonical form.

Low-rank perturbations arise in applied problems like...

- Structural modification of dynamical/vibrating systems (*pole-zero assignment*).
- Frequency compensation in electrical circuits.
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Low rank and genericity

What is the meaning of low rank and genericity ?

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The "classical" result for matrices

Theorem [Hörmander & Mellin, 1994], [Moro & Dopico, 2003]

$A \in \mathbb{C}^{n\times n}$, $\lambda_0 \in \sigma(A)$, $g = \dim \mathrm{Nul}\,(A - \lambda_0 I)$.

For generic perturbations $B \in \mathbb{C}^{n\times n}$ with $\operatorname{rank} B < g$:

The Jordan blocks of $A + B$ at λ_0 are the $g - \operatorname{rank} B$ **smallest** Jordan blocks of A at λ_0.

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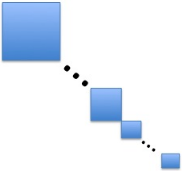

$J_{\lambda_0}(A)$

(g blocks)

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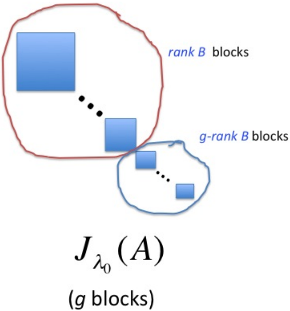

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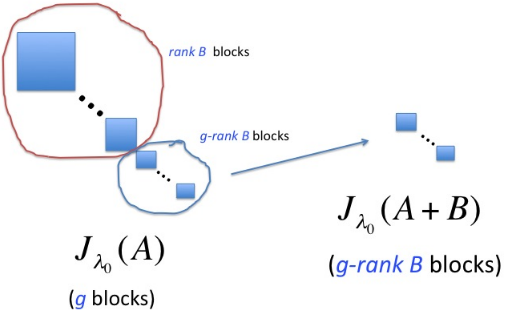

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Generic: $B \in \mathcal{M}_r \cap (\mathbb{C}^{n\times n} \setminus \mathcal{C})$

$\mathcal{M}_r = \{\text{matrices with rank} \leq r\}$

$\mathcal{C}$ an algebraic set in $\mathbb{C}^{n^2} \equiv \mathbb{C}^{n\times n}$

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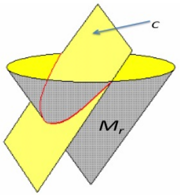

Definition of genericity

Generic subset of $\mathbb{C}^m$: $\mathbb{C}^m \setminus \mathcal{C}$, with $\mathcal{C}$ an algebraic set.

Problem: $\mathcal{M}_r$ is an algebraic set...but not irreducible !!! ($r+1$ irreducible components [D. & Dopico, 2008])

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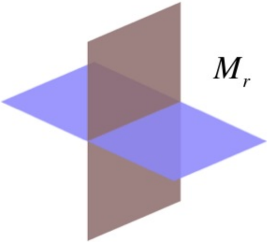

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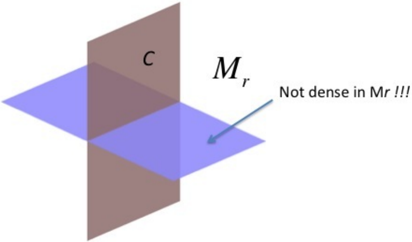

Definition of low rank

☞ For **matrices** $\rightsquigarrow$ A (unperturbed), B (perturbation), $A+B$ (perturbed):

$\text{rank}\, B < \dim \text{Nul}(A - \lambda_0 I) \Longrightarrow \lambda_0 \in \sigma(A+B)$

☞ For **regular pencils** $\rightsquigarrow$ $A_0 + \lambda A_1$ (unperturbed), $B_0 + \lambda B_1$ (perturbation), $A_0 + B_0 + \lambda(A_1 + B_1)$ (perturbed)

- **1st approach**: $\text{rank}(B_0 + \lambda_0 B_1) < \dim \text{Nul}(A_0 + \lambda_0 A_1) \Longrightarrow \lambda_0 \in \sigma(A_0 + B_0 + \lambda(A_1 + B_1))$
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Change of the WCF

$g = \dim \mathrm{Nul}(A_0 + \lambda_0 A_1)$

$n_1 \geq \ldots \geq n_g$: sizes of Jordan blocks at λ_0 in $A_0 + \lambda A_1$

Theorem [D., Dopico & Moro, 2008]

Set

$$\rho_0 = \operatorname{rank}(B_0 + \lambda_0 B_1), \qquad \rho_1 = \operatorname{rank} B_1, \qquad \text{and} \qquad r := \rho_0 + \rho_1.$$

If $\rho_0 < g$ then **generically** the Jordan blocks at λ_0 in $A_0 + B_0 + \lambda(A_1 + B_1)$ are obtained by **removing the first** r terms in the list:

$$n_1, \ldots, n_g, \overbrace{1, \ldots, 1}^{\rho_1}$$

☞ **Different** from the generic behavior for matrices (some 1×1 additional blocks may appear) !!!!

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Generic pencils with $\mathrm{nrank} - r$???

The way we had constructed the low-rank perturbations:

$$B_0 + \lambda B_1 = \underbrace{B_0 + \lambda_0 B_1}_{\text{rank}-\rho_0} + (\lambda - \lambda_0) \underbrace{B_1}_{\text{rank}-\rho_1}, \qquad r := \rho_0 + \rho_1$$

provided that $r < g \leq n$, satisfy (generically):

$$\mathrm{nrank}(B_0 + \lambda B_1) = \rho_0 + \rho_1 := r,$$

but...

Does not give generic pencils with $\mathrm{nrank} - r$!!!!!!!

Hint: $B_0 + \lambda B_1$ above has λ_0 as eigenvalue (with geometric multiplicity ρ_1)

(generic nrank-deficient pencils **do not have eigenvalues at all**)

Question: How to **generate** generic pencils with $\mathrm{nrank} - r$????

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Genericity and new approach

☞ **Generic set in $\mathcal{M}_r$:** **Dense open subset** of $\mathcal{M}_r$

Approach:

(1) $\mathcal{M}_r = \mathcal{C}_0 \cup \cdots \cup \mathcal{C}_r$

(2) For each $s = 0, 1, \ldots, r$:

$\Phi_s : \mathbb{C}^m \longrightarrow \mathcal{C}_s$, with $\mathcal{C}_s = \Phi_s(\mathbb{C}^m)$ **KEY**

Analyze what happens in: $\Phi(\mathbb{C}^m \setminus G_s)$, with G_s an algebraic set.

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Construction of pencils with fixed nrank – r

For each $s = 0, 1, \ldots, r$, set:

$$\mathcal{C}_s := \Big\{ \underbrace{v_1(\lambda)}_{\text{deg}=0} w_1(\lambda)^T + \cdots + \underbrace{v_s(\lambda)}_{\text{deg}=0} w_s(\lambda)^T + v_{s+1}(\lambda) \underbrace{w_{s+1}(\lambda)^T}_{\text{deg}=0} + \cdots + v_r(\lambda) \underbrace{w_r(\lambda)^T}_{\text{deg}=0} \Big\}$$

($v_i(\lambda), w_j(\lambda)$ are polynomial vectors with deg $v_i(\lambda), w_j(\lambda) \leq 1$).

Then:

$$\mathcal{M}_r = \mathcal{C}_0 \cup \mathcal{C}_1 \cup \cdots \cup \mathcal{C}_r \,.$$

Define a “coefficient map" (**surjective**):

$$\Phi_s : \mathbb{C}^{3rn} \longrightarrow \mathcal{C}_s$$

that assigns the coefficients of $v_i(\lambda), w_j(\lambda)$, for $i, j = 1, \ldots, r$.

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Guess: $\mathcal{C}_0, \ldots, \mathcal{C}_r$ are the irreducible components of $\mathcal{M}_r$ (we have proved it for $r = 1$).

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Change of the WCF revisited

$g = \dim \mathrm{Nul} P(\lambda_0)$

$n_1 \geq \ldots \geq n_g$: partial multiplicities of $P(\lambda)$ at λ_0

$\mathcal{M}_r = \mathcal{C}_0 \cup \mathcal{C}_1 \cup \cdots \cup \mathcal{C}_r$.

$\Phi_s : \mathbb{C}^{3rn} \longrightarrow \mathcal{C}_s$

Theorem

For each $s = 0, 1, \ldots, r$, there is a generic set $G_s \subseteq \mathbb{C}^{3rn}$ such that, for all $B_0 + \lambda B_1 \in \Phi_s(G_s)$, the Jordan blocks of $A_0 + B_0 + \lambda(A_1 + B_1)$ at λ_0 have sizes $n_{r+1} \geq \ldots \geq n_g$.

The largest r blocks dissappear !!!!!

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Theorem

There is a generic (dense open) set $G \subseteq \mathcal{M}_r$ such that, for all $B_0 + \lambda B_1 \in G$, the Jordan blocks of $A_0 + B_0 + \lambda(A_1 + B_1)$ at λ_0 have sizes $n_{r+1} \geq \ldots \geq n_g$.

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Outline

Singular pencils and regular matrix polynomials

Generic behavior also know for:

- Smith form of **regular matrix polynomials** (1st approach).
- Kronecker Canonical Form of singular matrix pencils (under the assumption that the **perturbed** pencil is **still singular** !!).

Structured perturbations

☞ Slightly different behavior due to the restrictions imposed by the structure.

Known results for:

- *J*-Hamiltonian matrices (rank-1 perturbations) [Mehl, Mehrmann, Ran, & Rodman, 2011]
- Selfadjoint matrices and sign characteristics (rank-1 perturbations) [Mehl, Mehrmann, Ran, & Rodman, 2012]
- Symplectic, Orthogonal, and Unitary matrices matrices (rank-1 perturbations) [Mehl, Mehrmann, Ran, & Rodman, 2014]
- *J*-Hamiltonian and *H*-symmetric (real) and sign characteristic (rank-1 perturbations) [Mehl, Mehrmann, Ran, & Rodman, submitted 2014]
- *T*-alternating, *T*-palindromic, and Symmetric pencils (rank-1 perturbations) [Batzke, 2014]
- *H*-selfadjoint, *J*-Hamiltonian matrices (rank-r perturbations) [Batzke, Mehl, Ran, & Rodman, submitted 2015]

Outline

Conclusions

- We have presented a description for the generic change of the WCF of regular matrix pencils under low rank perturbations.
- The way how these perturbations are **constructed** is important: we use a decomposition of $\mathcal{M}_r$ into $r+1$ subsets (irreducible components ???) that **"parameterize"** $\mathcal{M}_r$.
- The meaning of genericity has been analyzed (related to the construction of the perturbations !!).
- Still much work to be done: generic rank-r perturbations of matrix polynomials / structured pencils; allow for singular perturbed pencils, (low rank) distance to singularity, ...

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